DIFFERENTIATE-AND-FIRE TIME-ENCODING OF FINITE-RATE-OF-INNOVATION SIGNALS

Abijith Jagannath Kamath and Chandra Sekhar Seelamantula
Department of Electrical Engineering, Indian Institute of Science, Bangalore
{abijithj,css}@iisc.ac.in

1. INTRODUCTION

- Time-encoding or *event-driven sampling* is an alternative paradigm to Shannon sampling.
- We propose a differentiate-and-fire time-encoding machine (DIF-TEM) inspired by the *magnocellular pathway* in the human visual system.
- We propose kernel-based time-encoding of FRI signals with DIF-TEM via Fourier-domain analysis.

2. DIFFERENTIATE-AND-FIRE TIME-ENCODING MACHINE

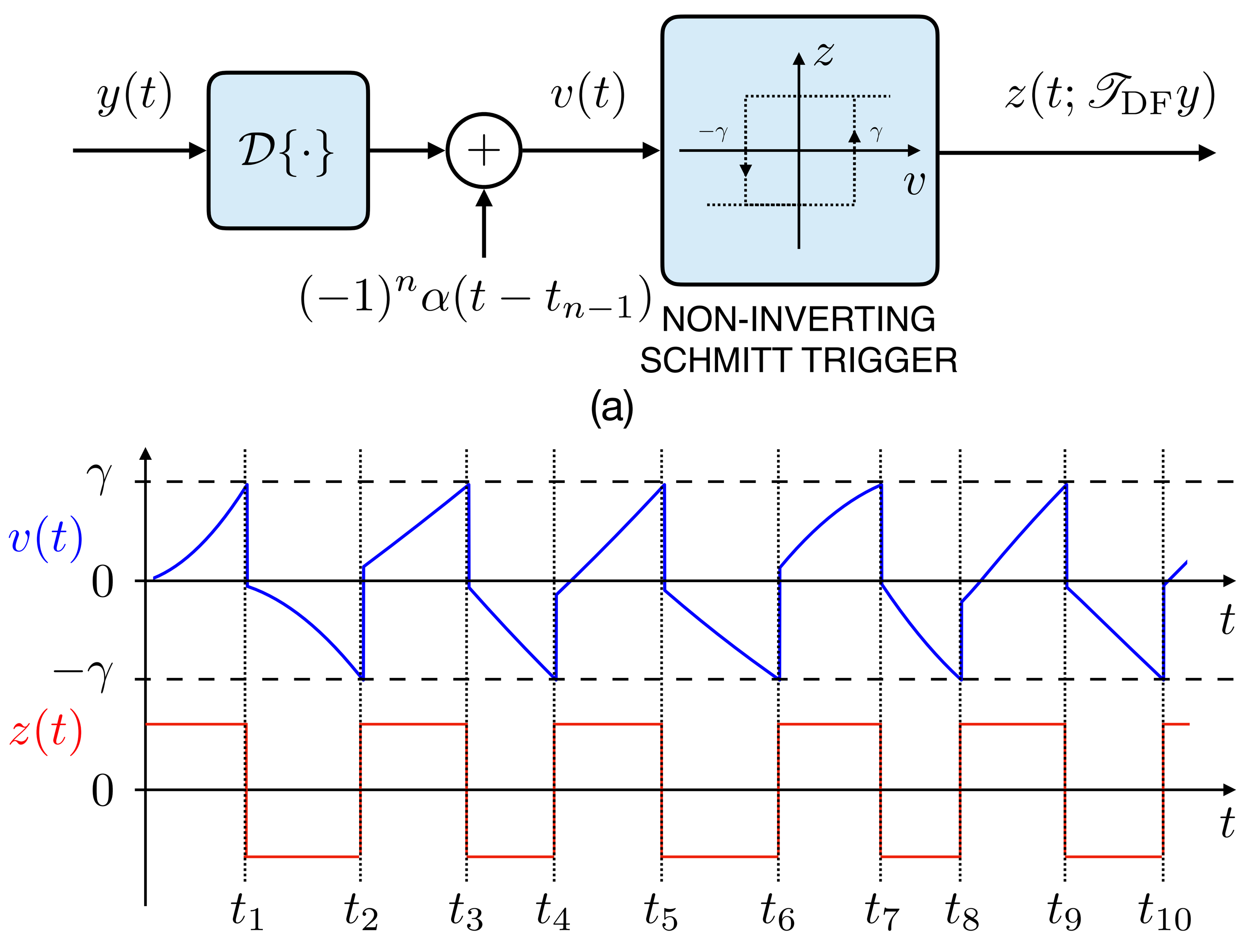

Fig 1: (a) A DIF-TEM with linear bias and a Schmitt trigger with threshold γ; (b) the sum $v(t)$ of the differentiated signal and the bias, and the output bilevel signal $z(t)$ that transitions at the trigger times $\mathscr{T}_{\mathrm{DF}}y$.

- **Lemma** *(t-transform):* Let $y \in \mathcal{C}^1(\mathbb{R})$. The output $\mathscr{T}_{\mathrm{DF}}y = \{t_n\}_{n\in\mathbb{Z}}$ satisfies

$$(\mathcal{D}y)(t_n) = (-1)^{n+1}(\gamma - \alpha(t_n - t_{n-1})).$$

- **Corollary** *(Sampling sets of DIF-TEM):* Let $y \in \mathcal{C}^1(\mathbb{R})$ with $\|Dy\|_\infty \leq \beta$. The output $\mathscr{T}_{\mathrm{DF}}y = \{t_n\}_{n\in\mathbb{Z}}$ satisfies

$$d(\mathscr{T}_{\mathrm{DF}}y) \doteq \sup_{n\in\mathbb{Z}} |t_n - t_{n-1}| \leq \frac{\gamma+\beta}{\alpha}.$$

3. TIME-ENCODING OF FRI SIGNALS

- Consider the T-periodic FRI signal $x \in L^2([0,T[)$

$$x(t) = \sum_{m\in\mathbb{Z}}\sum_{k=0}^{K-1} c_k\varphi(t-\tau_k-mT) \quad (1)$$

- The Fourier coefficients of $x(t)$

$$\hat{x}_m = \frac{1}{T}\hat{\varphi}(m\omega_0)\sum_{k=0}^{K-1} c_k e^{-\mathrm{j}\omega_0 m\tau_k},\ \omega_0 = \frac{2\pi}{T} \quad (2)$$

- $\{c_k\}_{k=0}^{K-1}$ and $\{\tau_k\}_{k=0}^{K-1}$ can be recovered using $\geq 2K+1$ measurements using Prony's method [1]

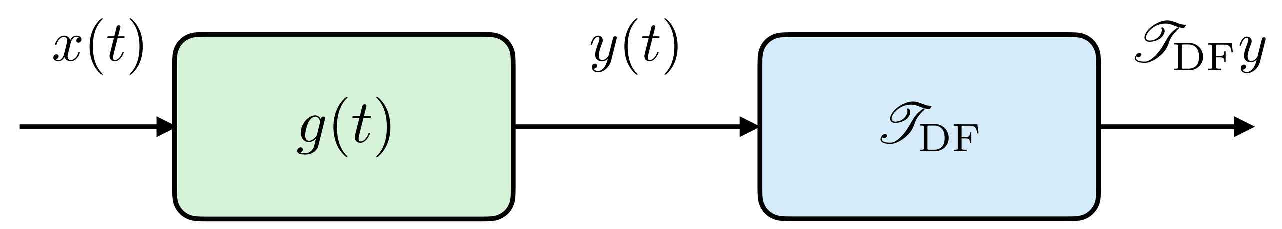

Fig 2: Kernel-based time-encoding of using a sampling kernel that satisfies the alias-cancellation conditions [2] and DIF-TEM.

- The filtered signal and measurements

$$y(t) = \sum_{m\in\mathbb{Z}} \hat{x}_m\hat{g}(m\omega_0)e^{\mathrm{j}\omega_0 mt} = \sum_{m=-M}^{M}\hat{x}_m e^{\mathrm{j}\omega_0 mt}$$
$$\implies y_n \doteq (\mathcal{D}y)(t_n) = \sum_{m=-M}^{M}\hat{x}_m\cdot(\mathrm{j}\omega_0 m)\cdot e^{\mathrm{j}\omega_0 mt_n} \quad (3)$$

- **Proposition** *(Sufficient condition for perfect recovery):* Let the FRI signal $x(t)$ be observed using a sampling kernel $g(t)$ that satisfies alias-cancellation conditions and a DIF-TEM with parameters $\alpha,\gamma > 0$. The time instants $\{t_n\}_{n=1}^{L} \subset \mathscr{T}_{\mathrm{DF}} \cap [0,T[$ constitute a sufficient representation of $x(t)$ if $L \geq 2K+1$ and the parameters of the DIF-TEM satisfy

$$\frac{\gamma + \|x * \mathcal{D}g\|_\infty}{\alpha} \leq \frac{T}{L}. \quad (4)$$

ACKNOWLWDGEMENTS

- Pratiksha Trust
- Institute of Eminence Funding
- Prime Minister's Research Fellowship

4. SIMULATION RESULTS

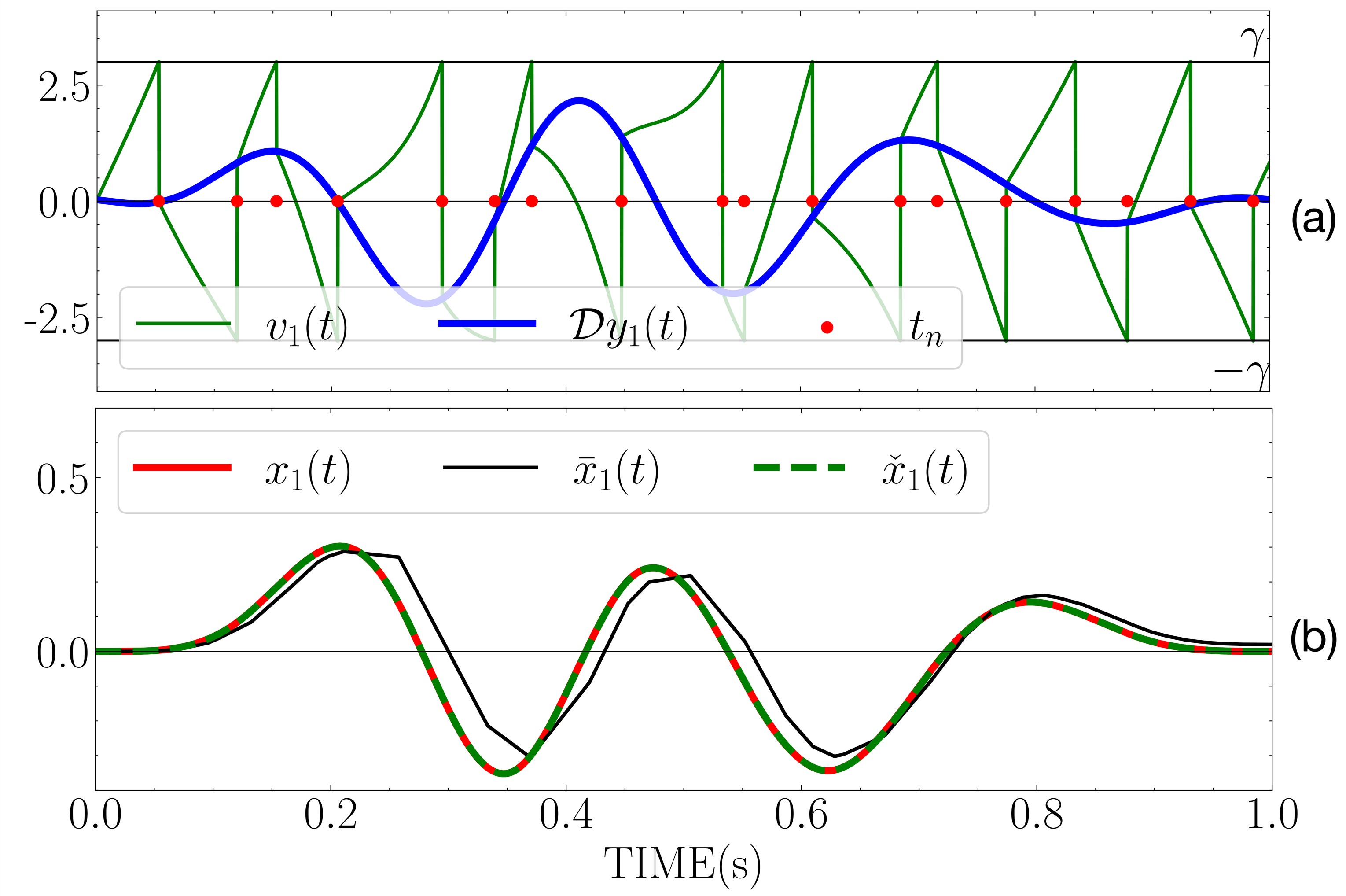

Fig 3: Reconstruction of cubic B-spline pulses from time-encoding using DIF-TEM: (a) derivative signal $\mathcal{D}y_1(t)$ and input to the Schmitt trigger $v_1(t)$; (b) input signal $x_1(t)$, reconstruction using proposed method $\tilde{x}_1(t)$ and using an integrator $\bar{x}_1(t)$.

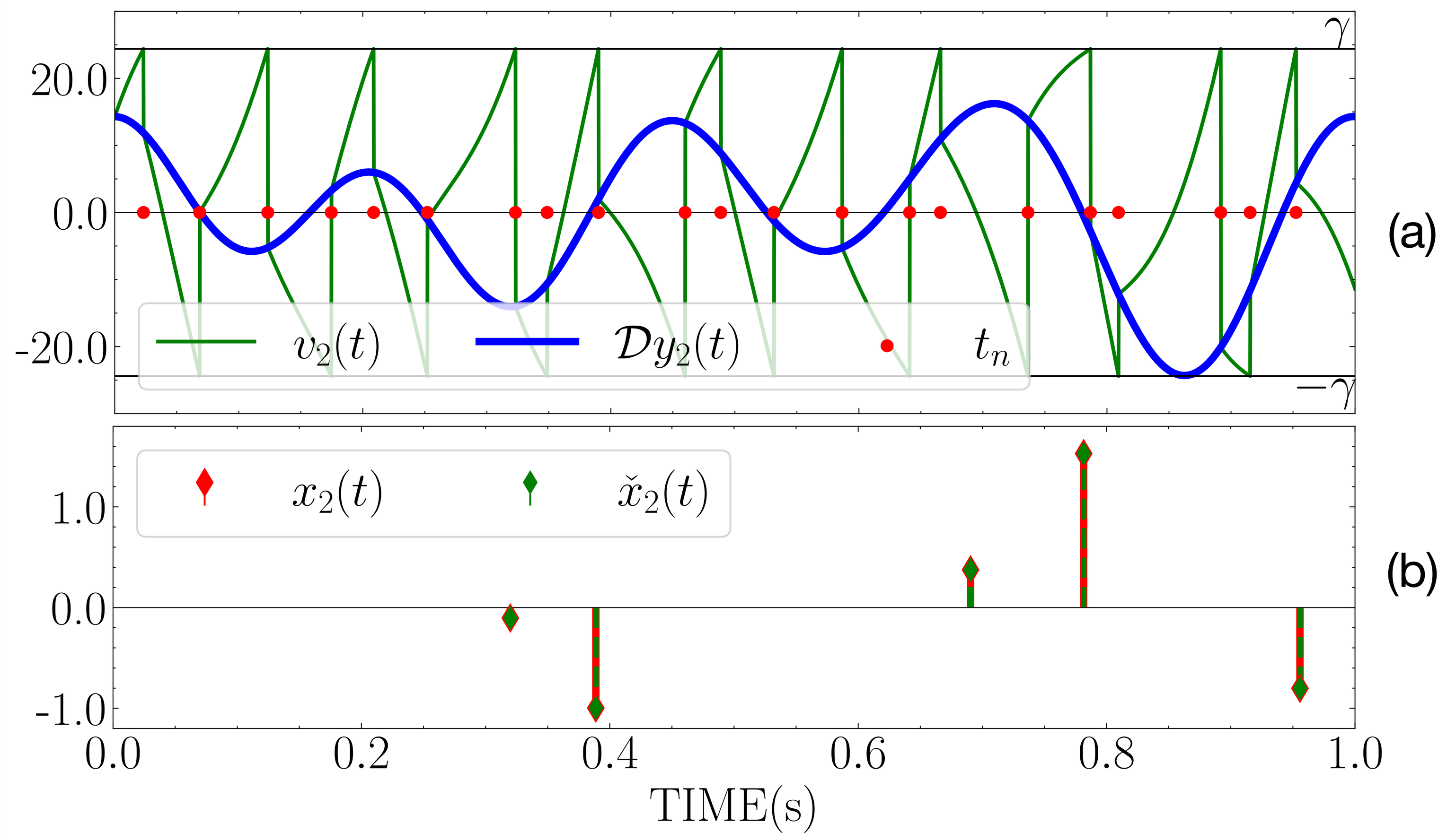

Fig 4: Reconstruction of Dirac impulses from time-encoding using DIF-TEM: (a) derivative signal $\mathcal{D}y_2(t)$ and input to the Schmitt trigger $v_2(t)$; (b) input $x_2(t)$ and reconstruction $\tilde{x}_2(t)$.

5. KEY REFERENCES

[1] Vetterli *et al.*, "Sampling signals with finite rate of innovation," IEEE Trans. Signal Process., 2002.
[2] S. Mulleti and C. S. Seelamantula, "Paley–Wiener characterization of kernels for finite-rate-of-innovation sampling," IEEE Trans. Signal Process., 2017.
[3] A. A. Lazar and L. Tóth, "Time encoding with an integrate-and-fire neuron with a refractory period," Neurocomputing, 2004
[4] D. Gontier and M. Vetterli, "Sampling based on timing: Time encoding machines on shift-invariant subspaces," Appl. Comput. Harmon. Anal., 2014.